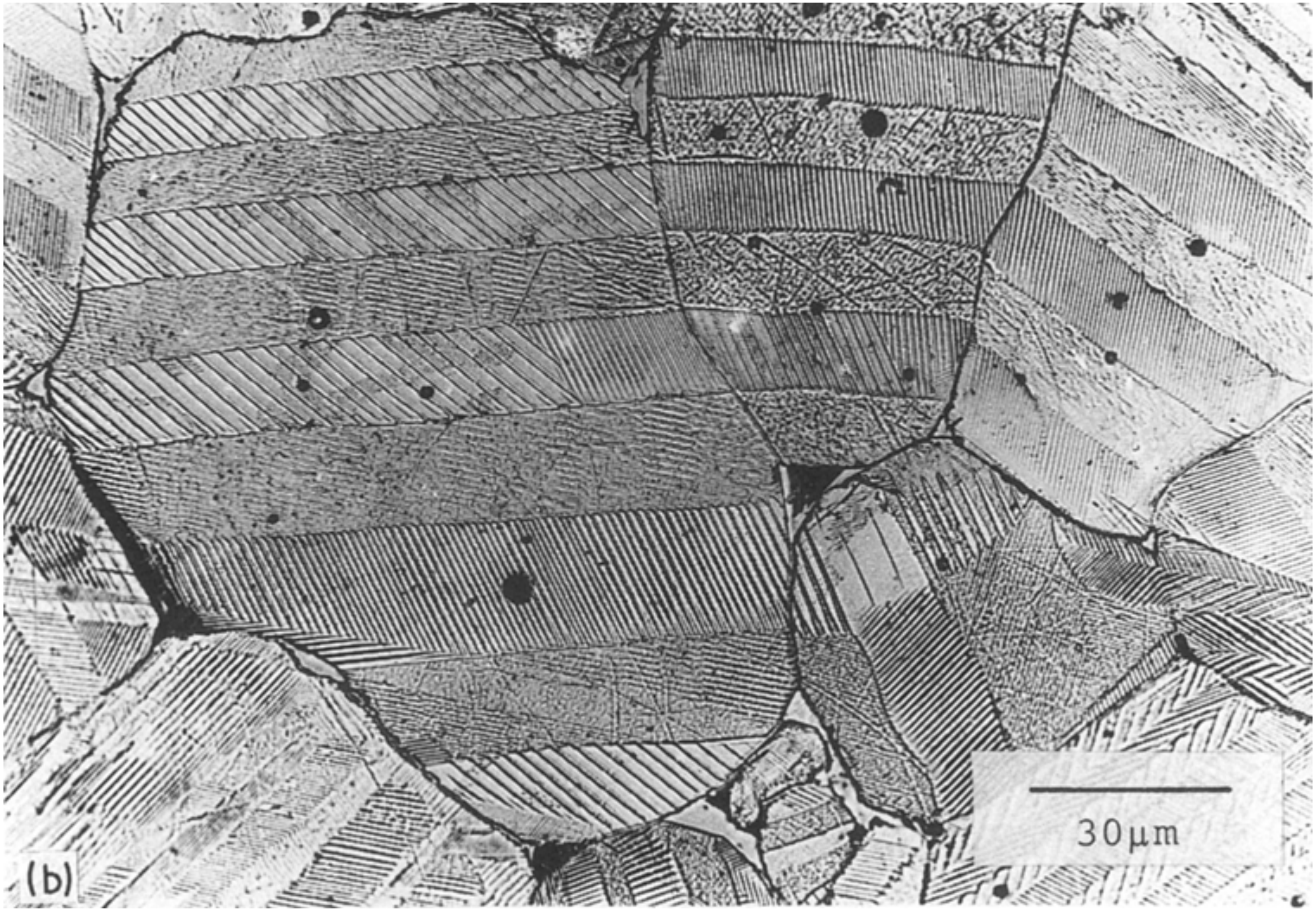
Polycrystal microstructures for more than two wells



$BaTiO_3$ ceramic: G. Arlt, J. Materials Science, 25 (1990) 2655-2666.

Consider a cubic-to-tetragonal transformation with

$$K = \bigcup_{i=1}^{3} SO(3)\mathbf{U}_i,$$

$$\mathbf{U}_1 = \text{diag}\,(\eta_2, \eta_1, \eta_1),\ \mathbf{U}_2 = \text{diag}\,(\eta_1, \eta_2, \eta_1),$$
$$\mathbf{U}_3 = \text{diag}\,(\eta_1, \eta_1, \eta_2).$$

Theorem

$\mathcal{E}$ contains a relatively open neighbourhood of $(\eta_1^2\eta_2)^{\frac{1}{3}}SO(3)$ in $\mathcal{D} := \{\mathbf{A} \in GL^+(3,\mathbb{R}) : \det \mathbf{A} = \eta_1^2\eta_2\}$.

Proof. $\mathcal{E}$ is isotropic and by Dolzmann & Kirchheim (2013) K^{qc} contains a relatively open neighbourhood of $(\eta_1^2\eta_2)^{\frac{1}{3}}\mathbf{1}$ in $\mathcal{D}$.

In fact, if the austenite is cubic and the transformation strain $\mathbf{U}$ is not a dilatation then K^{qc} always contains a nontrivial set of tetragonal wells (c.f. Bhattacharya (1992), B/Koumatos (2014)) and so $\mathcal{E}$ contains a relatively open neighbourhood of $(\det\mathbf{U})^{\frac{1}{3}}SO(3)$ in $\mathcal{D} := \{\mathbf{A} \in GL^+(3,\mathbb{R}) : \det\mathbf{A} = \det\mathbf{U}\}$. Hence, for example, we have a nontrivial $\mathcal{E}$ for cubic to orthorhombic transformations.

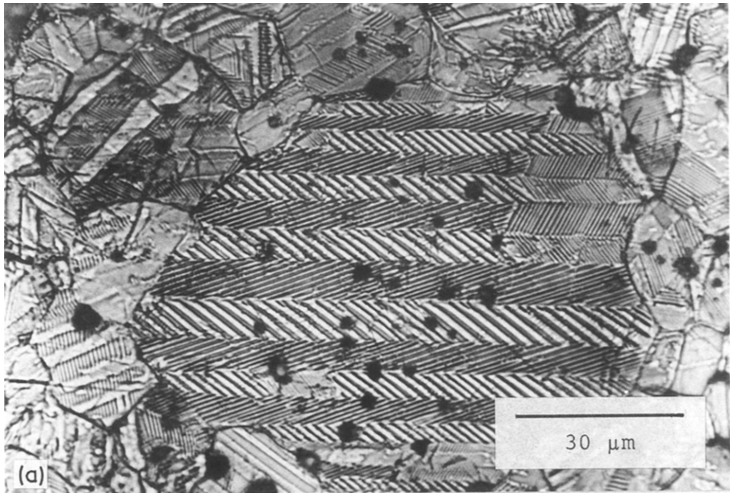
A related remark is:

Theorem There is no homogeneous gradient Young measure

$$\nu = \sum_{i=1}^{4} \lambda_i \delta_{\mathbf{A}_i}, \quad \lambda_i \geq 0, \sum_{i=1}^{4} \lambda_i = 1,$$

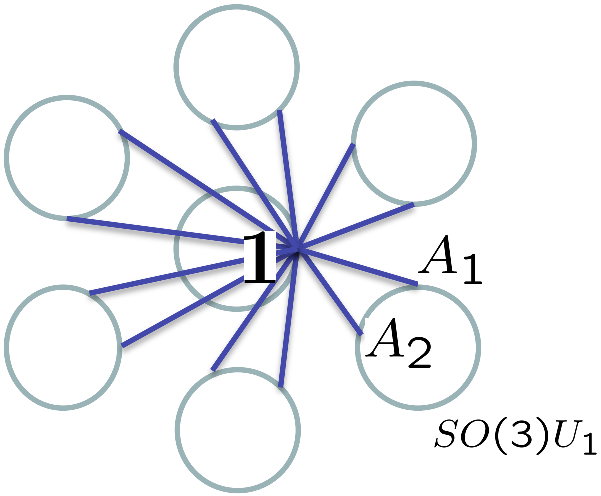
with $\mathbf{A}_i \in K$ and $\bar{\nu} = (\eta_1^2 \eta_2)^{1/3} \mathbf{1}$.

Arlt (1990). Microstructure with approximately four gradients in $BaTiO_3$.



Is the apparent conflict with experiment due to ignoring interfacial energy, or because the deformation is not a dilatation on the boundary?

Another issue (c.f. recent work of F. Della Porta) is whether all microstructures with $\text{supp}\,\nu_{\mathbf{x}} \subset K^{\text{qc}}\mathbf{R}_i$ for a.e. $\mathbf{x} \in \Omega_i$ are obtainable by a suitable path starting from the austenite.



$\text{rank}\,(\mathbf{A_i} - \mathbf{1}) = 1,$
$i = 1, \ldots, 12$

$\text{rank}\,(\mathbf{A_i} - \mathbf{A_j}) > 1,$
$i \neq j$

What is $\{\mathbf{A}_1, \ldots, \mathbf{A}_{12}\}^{\text{qc}}$?

$Ti_{76}Nb_{22}Al_2$
(T. Inamura)
cubic to
orthorhombic,
$\lambda_2 = 1$