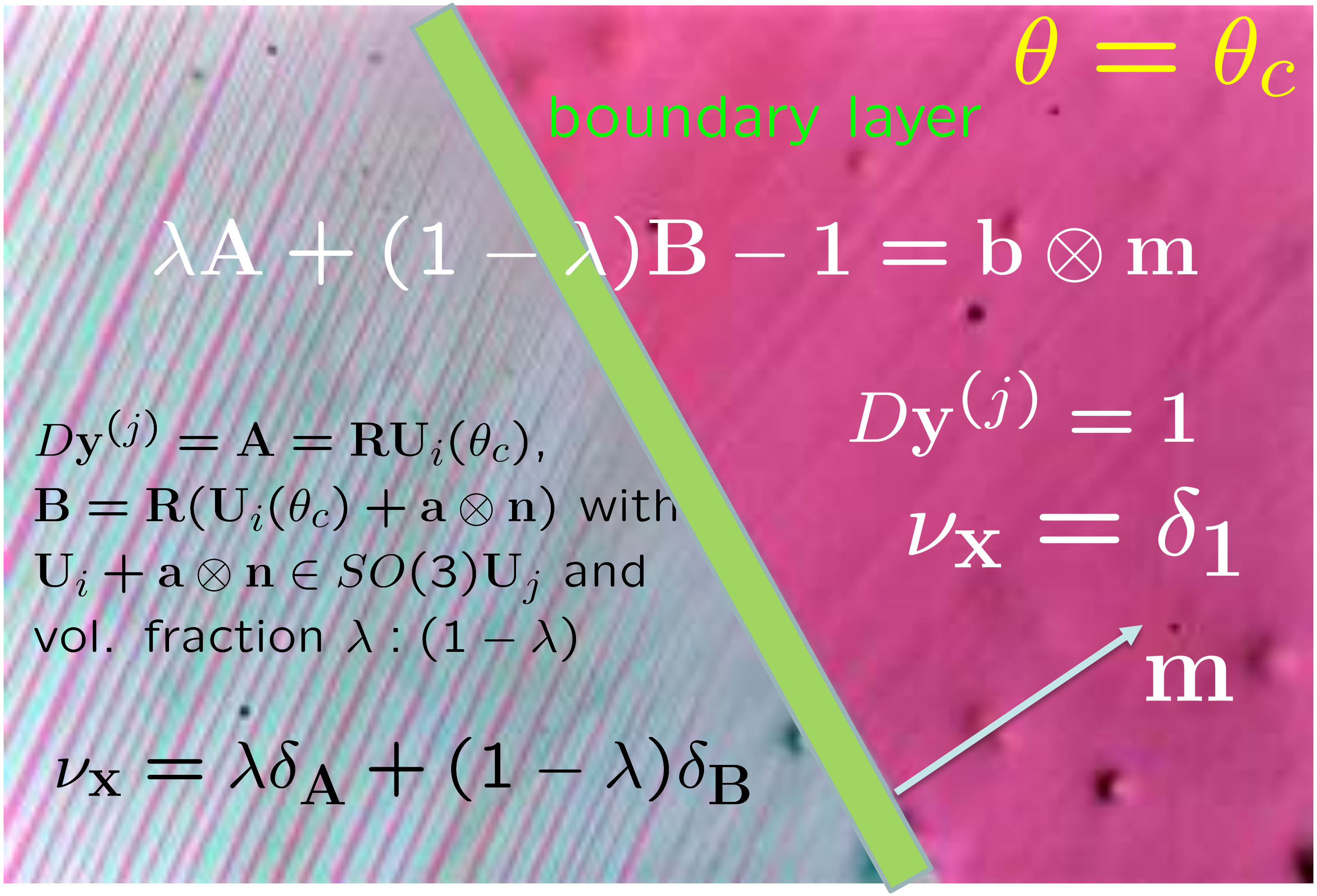
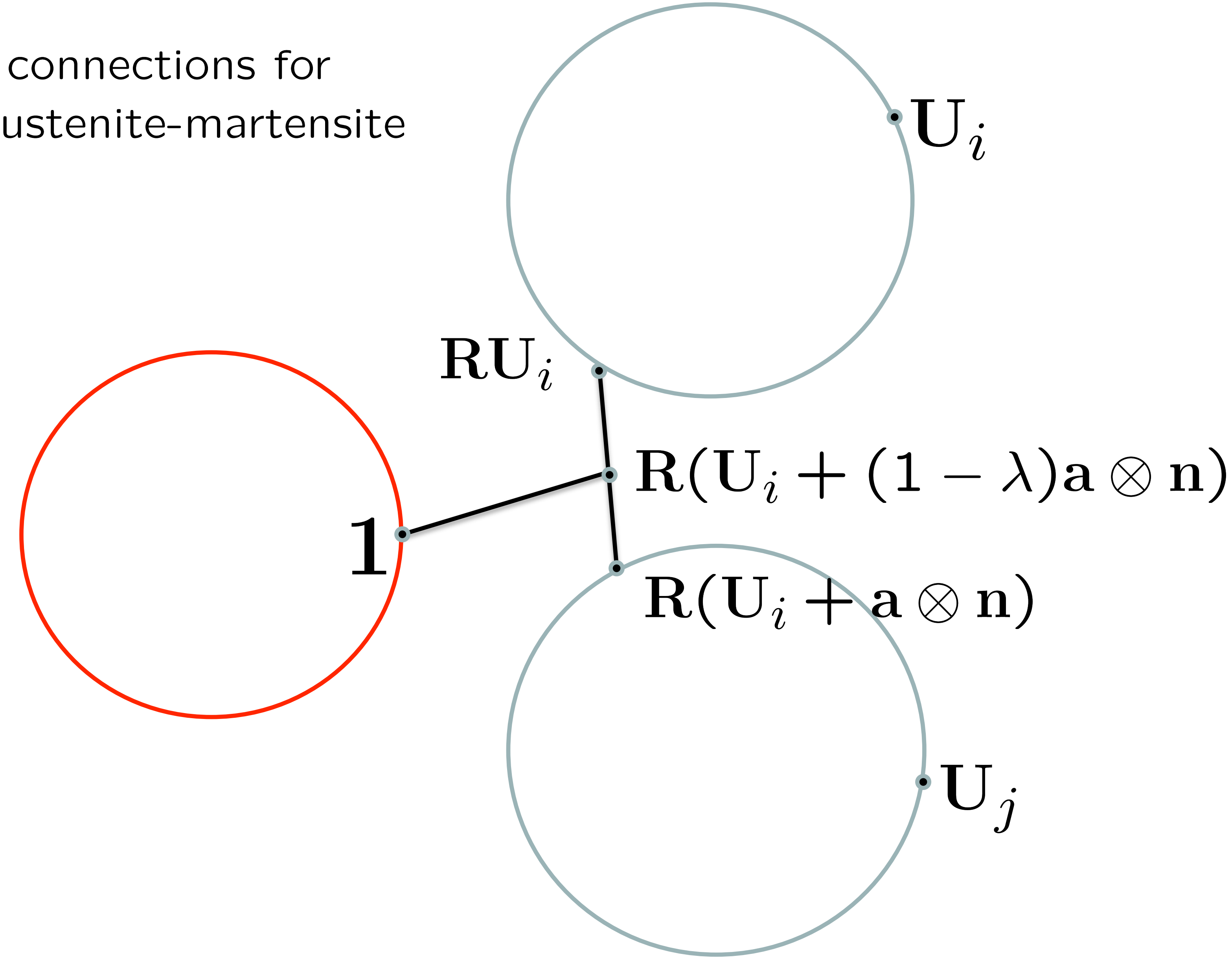


(Classical) austenite-martensite interface in CuAlNi (courtesy C-H Chu and R. D. James)

Rank-one connections for classical austenite-martensite interface



We have to solve

$$\mathbf{R}(\mathbf{U}_i + (1-\lambda)\mathbf{a}\otimes\mathbf{n}) - \mathbf{1} = \mathbf{b}\otimes\mathbf{m}$$

for $\mathbf{R} \in SO(3), \lambda \in [0,1]$ and $\mathbf{b}, \mathbf{m} \in \mathbb{R}^3$.

The solutions (JB/James 1987) give the formulae of the *crystallographic theory of martensite* (Wechsler, Lieberman & Read 1953)

Let $\delta^* = \mathbf{a} \cdot \mathbf{U}_i(\mathbf{U}_i^2 - \mathbf{1})^{-1}\mathbf{n}$.

Case 1. If $\mathbf{U}_i$ does not have an eigenvalue 1 then there is a solution iff $\delta^* \le -2$ and

$$\operatorname{tr} \mathbf{U}_i^2 - \det \mathbf{U}_i^2 - 2 + \frac{1}{2\delta^*}|\mathbf{a}|^2 \ge 0,$$

and if $\delta^* < -2$ there are exactly four solutions

$$(\mathbf{R}_1, \lambda^*, \mathbf{b}_1^+ \otimes \mathbf{m}_1^+), \quad (\mathbf{R}_2, \lambda^*, \mathbf{b}_1^- \otimes \mathbf{m}_1^-),$$
$$(\mathbf{R}_3, 1 - \lambda^*, \mathbf{b}_2^+ \otimes \mathbf{m}_2^+), \quad (\mathbf{R}_4, 1 - \lambda^*, \mathbf{b}_2^- \otimes \mathbf{m}_2^-),$$

where $\lambda^* = \frac{1}{2}\left(1 - \sqrt{1 + \frac{2}{\delta^*}}\right)$.

Case 2. There are solutions for *every* $\lambda \in [0,1]$ iff the following *cofactor conditions*

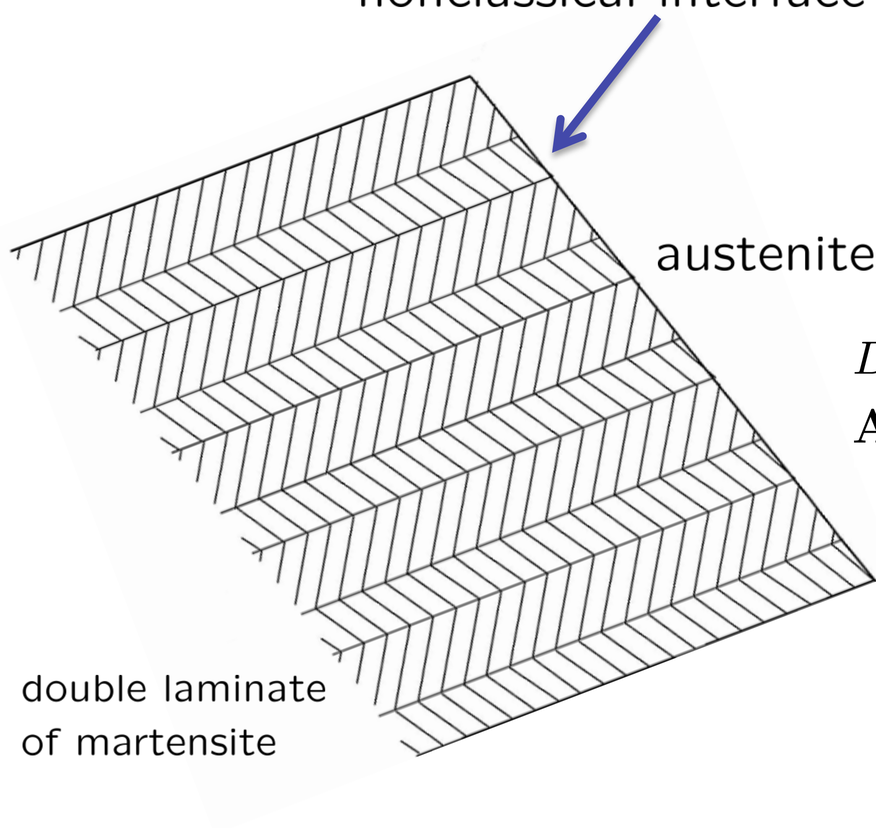
$\mathbf{U}_i$ has middle eigenvalue 1

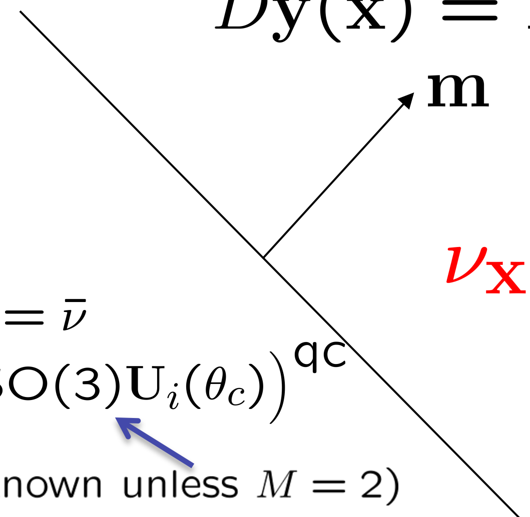
$\mathbf{a} \cdot \mathrm{cof}\,(\mathbf{U}_i^2 - \mathbf{1})\mathbf{n} = 0,$

$\mathrm{tr}\,\mathbf{U}_i^2 - \det \mathbf{U}_i^2 - \frac{|\mathbf{a}|^2}{4} - 2 \geq 0$

hold.

But why (cf JB/Carstensen 1997) should the martensitic microstructure be a simple laminate, rather than something more complicated, such as a double laminate?





$$D\mathbf{y}(\mathbf{x}) = \mathbf{1}$$

$$\nu_{\mathbf{x}} = \delta_{\mathbf{1}}$$

$$D\mathbf{y}(\mathbf{x}) = \mathbf{A} = \bar{\nu}$$

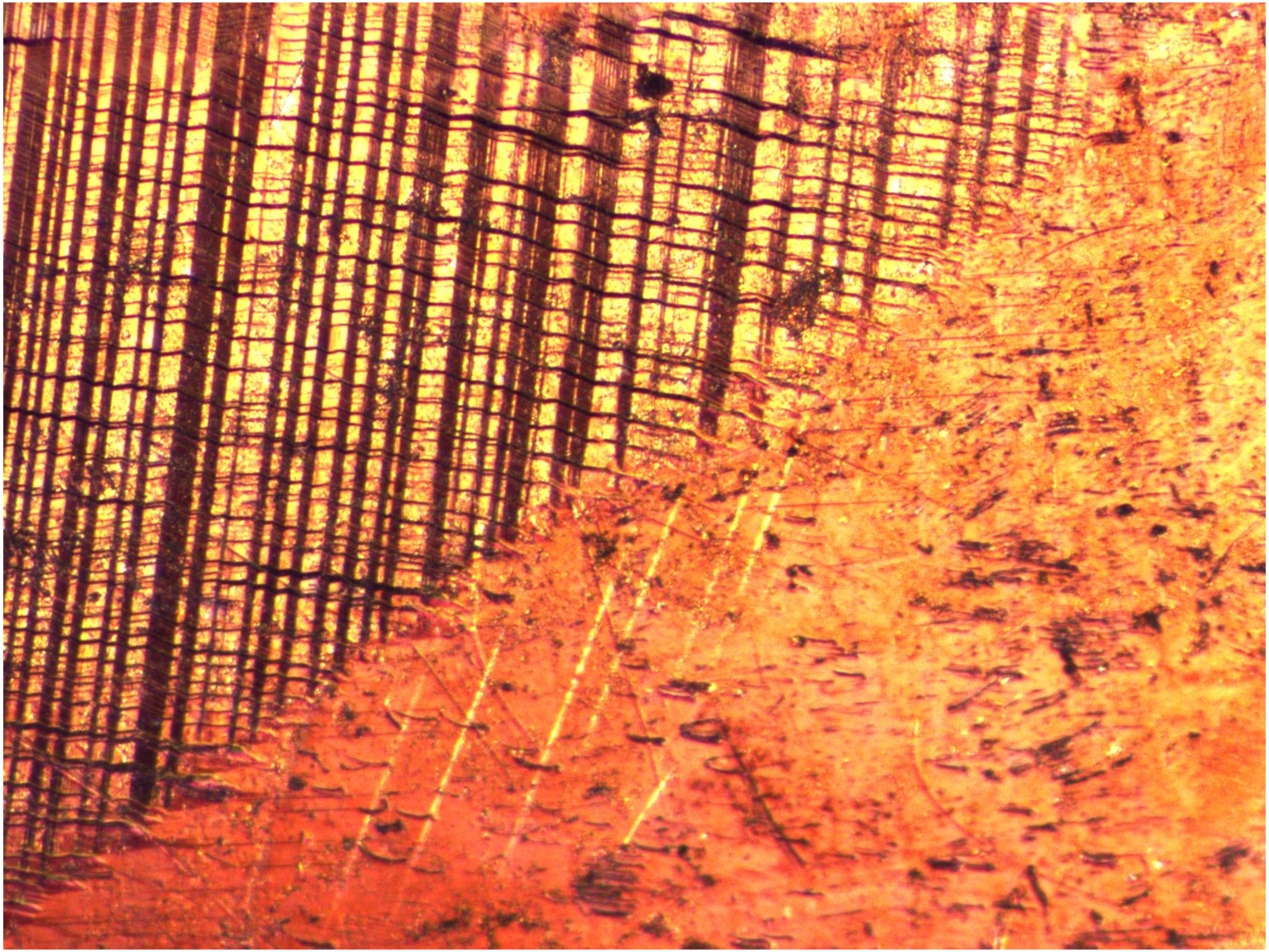
$$\mathbf{A} \in \left(\bigcup_{i=1}^{M} SO(3)\mathbf{U}_i(\theta_c)\right)^{qc}$$

(unknown unless $M = 2$)

$$\nu_{\mathbf{x}} = \nu$$

$$\text{supp}\,\nu \subset \bigcup_{i=1}^{M} SO(3)\mathbf{U}_i(\theta_c)$$

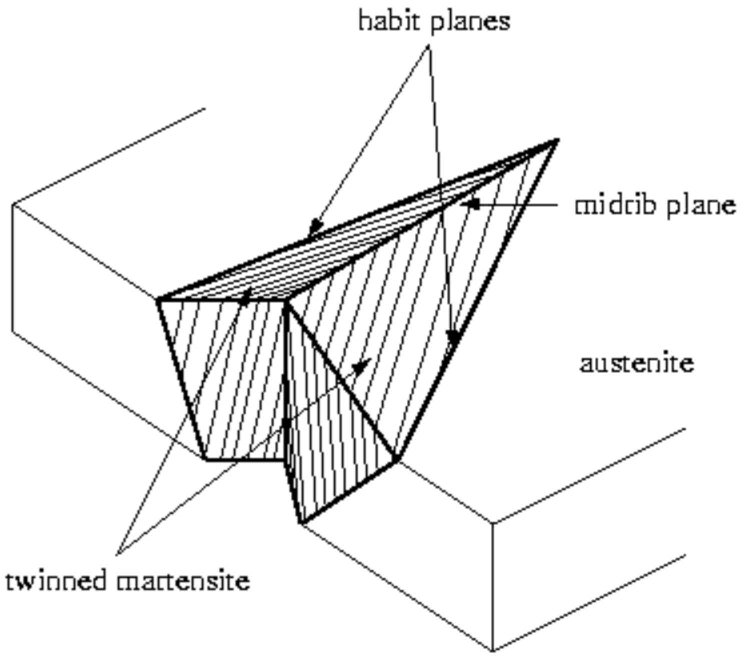
$$\mathbf{A} = \mathbf{1} + \mathbf{b} \otimes \mathbf{m}$$

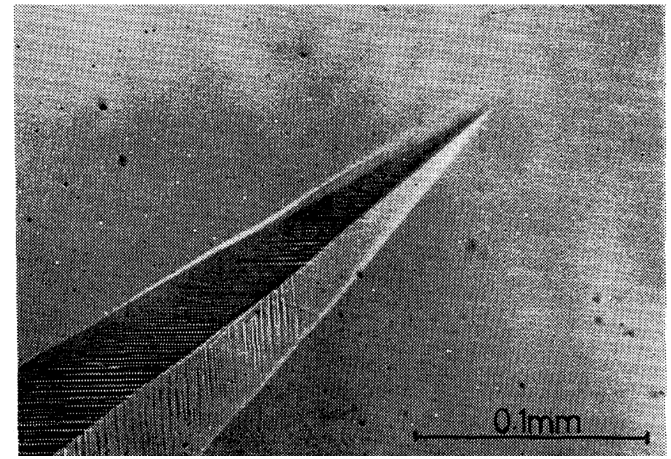


Nonclassical austenite-martensite interface in CuAlNi (H. Seiner)

Special compositions and the discovery of low hysteresis alloys.

1. The wedge microstructure (Bhattacharya 1991)





Wedge microstructure in CuAlNi
Otsuka & Shimizu (1969)

Microstructure supported on energy wells impossible for cubic-to-tetragonal, possible for cubic to orthorhombic iff the eigenvalues α, β, γ of the transformation strain $\mathbf{U}_i(\theta_c)$ satisfy a special relation $f(\alpha, \beta, \gamma) = 0$, which holds to high accuracy for the actual compositions close to Cu-14.2wt.%Al-4.3wt.%Ni used in shape-memory alloys.

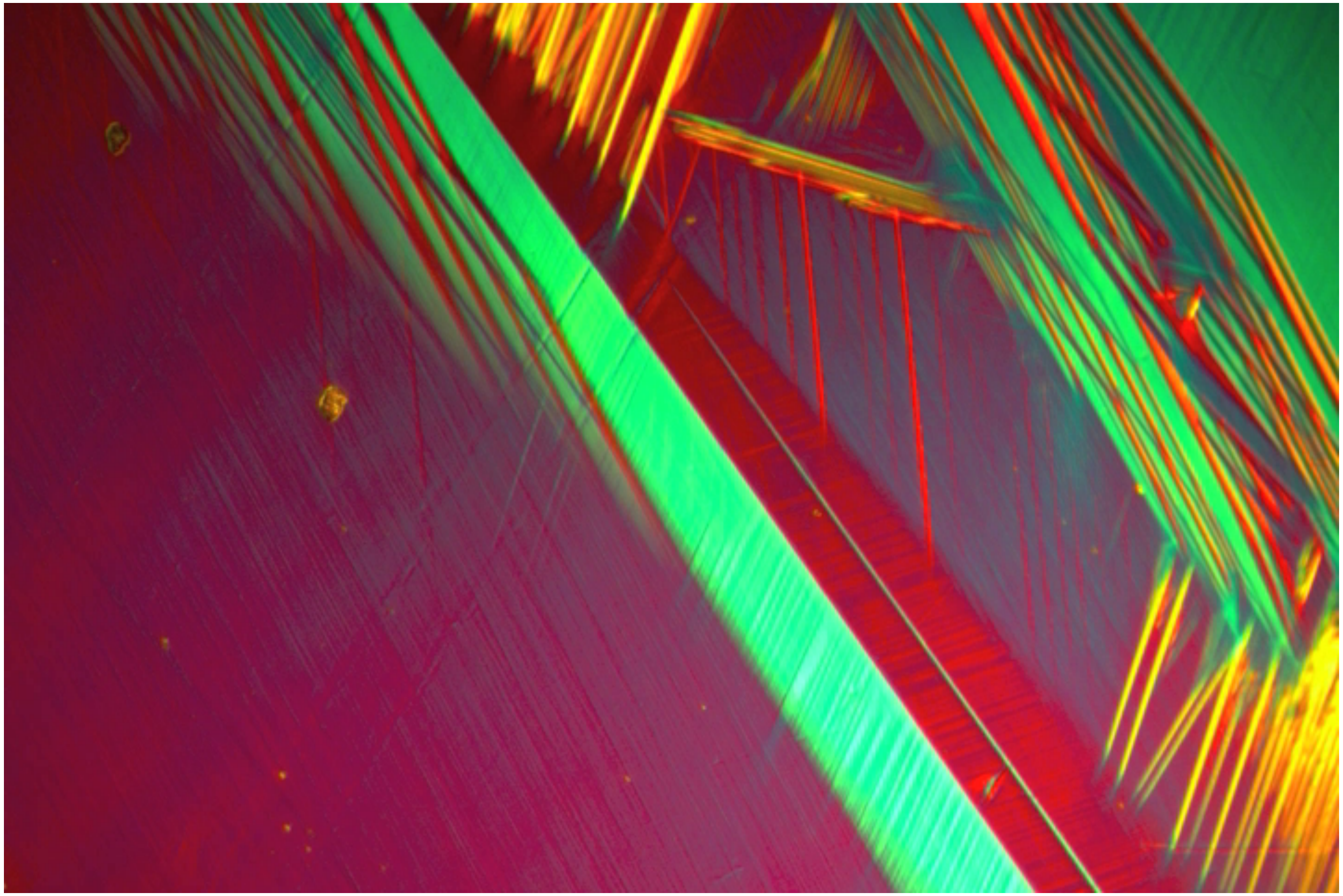
2. Ultra-low hysteresis alloys

James *et.al.* (2013) tuned the composition of a ZnAuCu alloy so that the cofactor conditions were very nearly satisfied, with dramatic results.

(i) the thermal hysteresis was reduced from typical values of $50^\circ - 70^\circ$C to about 2°C.

(ii) Material undamaged after thousands of thermal cycles (millions for a material discovered later by Quandt, Wuttig et al 2014).

(iii) During thermal cycling remarkable martensitic microstructures are observed that are completely different in each cycle.



$Zn_{45}Au_{30}Cu_2$ ultra-low hysteresis alloy Song, Chen, Dabade, Shield, James, 2013

'Moving mask' approximation analyzed by Della Porta (2018), who has also identified further conditions on the $\mathbf{U}_i$ allowing new microstructures, closely satisfied in this alloy.